Ordering Perceptions about Perceptual Order

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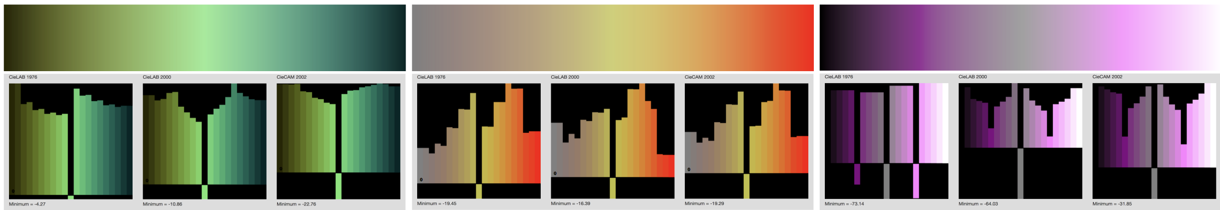

Figure 1: Top: Colormaps that provide counter examples to the hypotheses that monotonicity in an attribute implies intrinsic order. Left to right: Monotonic in *hue*, *saturation*, and *luminance*. Bottom: Local triangle side differences below zero indicate locations that violate intrinsic order. Images generated via [9].

Abstract

One of the most important properties that inherently defines a good colormap is perceptual order. In the literature, we find a wide range of recommendations and hypotheses regarding order. Properties such as monotonicity in luminance, saturation, or hue are/are not stated as necessary/sufficient to ensure perceptual order. In this paper, we gather the most common statements about perceptual order and, when possible, prove or disprove them.

Index Terms: Human-centered computing—Visualization—Visualization techniques—Colormapping; Human-centered computing—Visualization—Visualization design and evaluation methods

1 Introduction

Important properties of a colormap often depend on the task and data type. Discriminative power may be most important to highlight subtle features or differences in the data. Uniformity in a colormap allows a scientist to find features of interest regardless of where they may fall in the data.

Perceptual order is another important colormap characteristic and itself comes with varying attributes. Order can occur locally or globally. It can be directed or undirected. It can be considered an intrinsic property or, alternately, something that only has meaning in the context of a visible legend. Given the many facets of perceptual order, it is not surprising that colormapping literature is full of recommendations, exhortations, and conflicting advice on how best to achieve perceptual order when developing a colormap.

In this paper, we collect the many hypotheses relating to perceptual order from the body of related work. Where possible, we prove or disprove these hypotheses based on the mathematical framework developed in Bujack et al. [8]. We do not attempt to determine whether order is relevant or critical for a specific task or data. Our approach is agnostic to such specifics and simply seeks to mathematically validate or rule out the many hypotheses involving order that can be found in the color literature.

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2 Related Work

The importance of order for the design of a good colormap for data that is itself ordered has been stressed throughout literature. Sloan and Brown [27] state as far back as 1979 that a colormap should consist of colors that have an order that can be remembered easily.

Wainer and Francolini [29] stress the importance of order within a retinal variable. They consider size, length, density, and darkness to be ordered and point out that it is difficult to order colors solely by hue even though colors have a natural order in the spectrum. They also mention that saturation can be used to reflect order.

Trumbo [28] agrees that a good univariate colormap should have order in one of the retinal variables or their combinations, by which he refers to hue, saturation, and brightness.

Pizer at al. [18, 19] state that natural order can be achieved by increasing monotonically in brightness and each of the RGB components, such that the order of their intensities does not change throughout the colormap. They consider the grayscale and the heated body map as naturally ordered while a rainbow scale is not ordered, even if it is monotonic in brightness. They further suspect that a colormap that suffices natural order must at least be monotonic in brightness.

Ware [30] points out that a monotonic change in luminance is important to see the overall form of the data (qualitative task). On the other hand, he stresses the significance of non-monotonicity in at least one color-opponent channel, to read the underlying values of the data (quantitative task).

Levkowitz and Herman [15] disagree with Pizer in that monotonicity in brightness, RGB and their mutual order is not sufficient to guarantee order. They additionally require monotonicity in hue and constancy in relative saturation. Following these restrictions, they generate 72 colormaps transitioning from black to white and find that the longest one is 5.8 times the length of the grayscale.

Brewer and Harrower [6, 7, 12] suggest that, to avoid confounding attributes, perceptual progression should relate to progression in value. They use monotonicity in luminance, but not in saturation. Their maps show decreasing saturation towards both ends with a saturation maximum somewhere in the middle depending on hue to increase discriminative power. They allows changes in hue as long as they do not dominate over the lightness change and state that it is possible to go through the whole circle without violating this rule.

Rogowitz and her many collaborators [2, 14, 21–25] distinguish different tasks, data types, and data frequency and recommend colormap properties for each combination. For the isomorphic task, i.e., faithfully reflecting the structure of the underlying data, they recommend equal visual importance, perceptually even spacing, smoothness, and monotonically increasing luminance, saturation, or hue and warn about using the rainbow colormap.

Green [11] summarizes Bertin's principles [3], including that the grayscale is ordered and that monotonicity allows order but not quantitative comparisons. Green explicitly notes that though hue does not have a global order, it does over small ranges.

Rheingans [20] summarizes previously suggested rules. She agrees that the grayscale is ordered while the rainbow is not.

Borland and Taylor [5] focus on analyzing the flaws of the rainbow. They explicitly apply a thought experiment drawn from Ware [31] to illustrate the problem of ordering four colors drawn from the spectrum, but also recognize that humans are able to sort the hues correctly if the range is small. Further, they state that changes in luminance are a strong cue indicating order and that the grayscale and the heated body map are perceptually ordered.

Wijffelaars et al. [32] state that lightness produces order while monotonicity in saturation is not necessary, because the color scales suggested by Brewer [6,7] do not fulfill this criterion. They also mention that Brewer's colormaps do not increase linearly in CIELUV's luminance, but do not go as far as explicitly stating that this is not a necessary requirement, as they did for the saturation.

Moreland [17] also stresses the importance of order. He designed the red-blue diverging colormap that has replaced the rainbow as the default in the visualization tool ParaView [1].

Borland and Huber [4], on the other hand, have a more controversial opinion. They advocate for sacrificing traditional rules such as order if necessary to address the needs of the application scientist.

Bujack et al. [8] collect the different ways in which authors use the term order. They distinguish local vs. global, intuitive vs. easy to remember vs. legend-based and directed vs. undirected perceptual order and assign a mathematical formula to unambiguously define the terms when possible.

3 Hypotheses

As seen in the related work, the recommendations gathered here span a range of predictions for what is necessary and/or sufficient to ensure order. Almost all authors working on the topic agree that there is a strong correlation between monotonicity in luminance and order. In particular, a colormap should be monotonic in luminance to encode order and non-monotonic in luminance if the underlying data is not ordered [2,6,7,12,14,20–25]. Far more rarely do authors explicitly state that a specific property is necessary or sufficient. Not all statements rise to the level of a verifiable hypothesis. We summarize the statements that are assertive enough to be hypotheses.

- The grayscale (brightness, luminance) has perceptual order [5,11,18–20,27,29].
- Saturation has perceptual order [11,29].
- Hue has local order [5,11,28].
- Hue has no global order [5,11,12,18–20,28,31].
- Monotonicity in luminance is necessary for order [18,19].
- Monotonicity in luminance is sufficient for order [4,16,32].
- Monotonicity in luminance is not sufficient for order [15,18,19].
- Monotonicity in hue is not sufficient for order [15,18,19].
- Monotonicity in luminance, RGB, and their order is sufficient for natural order [18,19].
- Monotonicity in luminance, RGB, and their order, hue, and saturation is sufficient for natural order [15].
- Monotonicity in saturation is not necessary for perceptual order [12,18,19,32].
- Strict monotonicity in saturation is not necessary for perceptual order, but constancy [15].

4 Theory

In this section, we will attempt to prove or disprove the validity of the above hypotheses. Table 1 summarizes these hypotheses along with the results of our investigation. We begin by recapping the necessary theoretical foundations.

4.1 Foundations

For the concept of a colormap and the differences between colors to make sense, we assume that the **color space** is a 3D metric space that is path-connected.

A **path** between $x, y \in C$ is a continuous map $\gamma : [0,1] \to C$ with $\gamma(0) = x$ and $\gamma(1) = y$. The length of a path is defined as

$$L(f) = \sup_{0=t_0<\ldots<t_n=1} \sum_{i=1}^{n} d(\gamma(t_i) - \gamma(t_{i-1})) \tag{1}$$

The principle of *diminishing returns* [13] makes the space not convex, especially not a length metric space. That means that the length of a shortest path in general does not coincide with the distance between its endpoints. All Riemannian metric spaces are length metric spaces.

For the concepts of monotonicity in **hue, saturation, or luminance** to be defined, we assume that the latter three uniquely assign a real value to each point in the color space $h, s, l : C \to \mathbb{R}$. We say that a path through color space $\gamma : [0,1] \to C$ is **strictly monotonic** w.r.t. a real-valued function $f : C \to \mathbb{R}$ if

$$\forall t_i < t_j \in [0,1] : f(\gamma(t_i)) \lesseqgtr f(\gamma(t_j)). \tag{2}$$

Even though most authors do not explicitly specify whether they talk about monotonicity or strict monotonicity, we will use the concept of strict monotonicity throughout the paper and omit the word strict for brevity. Otherwise, a colormap that consists of one color only could always serve as a trivial counter example. Also, we will assume that all points that are chosen to evaluate order are at least one just noticeable difference (JND) [10] apart to avoid meaningless but generally possible counter examples.

We define the **luminance, saturation, and hue maps** as a colormap that monotonically changes in the named attribute and is constant w.r.t. to the other two. We will further follow Schrödinger's assumptions that the luminance and saturation maps lie on shortest paths in the color space [26]. Please note that the classical rainbow map does not fall into this definition of hue map because it varies in luminance. The luminance map is also called grayscale.

Figure 2: An example of how intuition can vary by culture (left to right): heated body colormap, German flag, Belgian flag.

We make use of the definitions and distinction of order from Bujack et al. [8]. In their framework, a colormap is considered to satisfy **order** if, given a set of colors picked from the colormap, everybody sorts them in the same way. They distinguish intuitive from legend-based, i.e., either the sorting is performed without, or with access to the legend; and local from global, i.e., the sample points are chosen either consecutively or arbitrarily distant. We will use the term *intrinsic* instead of intuitive because it describes the intrinsic property of a color space. Intuitive order as used by many authors cannot be captured in a single color space because intuition differs between individuals. For example, Fig. 2, due to the association with their flags, a German would likely agree that the

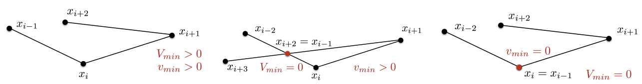


Figure 3: Illustration of local vs. global legend-based order, Equation (3) (left to right): local and global; local but not global; neither local nor global.

heated body colormap is ordered, while a Belgian would probably sort black-yellow-red.

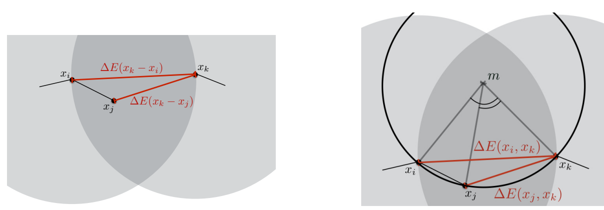
They correlate **legend-based order**, Fig. 3, to the invertibility of a colormap, i.e., a colormap suffices legend-based order if it does not pass through the same point twice and local legend-based order as long as two neighboring colors do not coincide:

$$\forall t_i \neq t_j \in [0,1] : \gamma(t_i) \neq \gamma(t_j). \tag{3}$$

Further, they assume a colormap suffices **intrinsic order** if the distance of the two outer colors is larger than their respective distances to the one in the middle. A visualization of this concept can be found in Fig. 4(a); the corresponding mathematical formulation is

$$\begin{aligned} &\forall t_i < t_j < t_k \in [0,1] : \\ \Delta E(\gamma(t_j),\gamma(t_i)) < \Delta E&(\gamma(t_k),\gamma(t_i)) > \Delta E(\gamma(t_k),\gamma(t_j)). \end{aligned} \tag{4}$$

We will adapt these **global** definitions (3), (4) throughout the paper. The **local** analogues are derived by setting $t_i = t_j - \Delta t, t_k = t_j + \Delta t$.



(a) (b)

Figure 4: (a): Intrinsic order based on the triangle side difference, Equation (4). The dark gray area suffices order w.r.t. the outer points. (b) Intrinsic order on partial hue maps as in Theorem 5.

4.2 Legend-Based Order

The next Theorem shows that all hypotheses that identify monotonicity as sufficient for local and global legend-based are true.

Theorem 1. *A colormap $\gamma : [0,1] \to C$ that suffices strict monotonicity w.r.t. a real-valued function $f : C \to \mathbb{R}$ suffices local and global legend-based order.*

Proof. From strict monotonicity follows that $\forall t_i \neq t_j \in [0,1] : f(\gamma(t_i)) \neq f(\gamma(t_j))$, which implies $\forall t_i \neq t_j \in [0,1] : \gamma(t_i) \neq \gamma(t_j)$ and therefore legend-based order in its local and global sense. □

Since the grayscale and a saturation map are shortest paths, the fact that they suffice local and global legend-based order follows from the fact that a shortest path is invertible.

Theorem 2. *A shortest path between two points suffices legend-based order.*

Proof. Assume that there is a shortest path that visits the same point twice, we can drop everything between these two visits and one of the two occurrences of that point and produce a path between the same endpoints. It follows from the identity of indiscernibles that this path is shorter, which contradicts the assumption. That means a shortest path must satisfy legend-based order. □

4.3 Intrinsic Order of Grayscale and Saturation Map

Since the grayscale and a saturation map are shortest paths, the question of whether they suffice local and global intrinsic order translates mathematically to the hypothesis that the shortest path between two points x_i, x_k lies within the ball of radius $\Delta E(x_i, x_k)$.

This hypothesis is true for any color space known to the authors. In a length metric space, which includes Riemannian spaces, the assertion also holds.

Theorem 3. *In a length metric space, the shortest path between two points x_i, x_k lies within the ball of radius $\Delta E(x_i, x_k)$.*

Proof. In a length metric space, the shortest path between x_i and x_k has length $\Delta E(x_i, x_k)$. If x_j lies on this path, then x_i, x_j, x_k is a segmentation of the path as in Equation 1 with length $\Delta E(x_i, x_j) + \Delta E(x_j, x_k)$. The length of this path is the supremum over all segmentations, which is of course not smaller than any segmentation, i.e. $\Delta E(x_i, x_k) \geq \Delta E(x_i, x_j) + \Delta E(x_j, x_k)$. From the positivity of the metric follows that the triangle distance difference holds. □

Even though the authors don't believe that a space with not ordered shortest paths will model human color vision well, theoretically, such a space could exist in a non-Riemannian setting. That means for the most general case of a path-connected metric space in which all distances are purely based on experiments, we cannot prove that the grayscale or the saturation maps are intrinsically ordered.

4.4 Intrinsic Order of Hue

There is an important difference between the saturation or luminance maps and the hue maps: the latter is periodic. A hue map that covers every hue necessarily has the same start and endpoint $\gamma(0) = \gamma(1)$. As a result, it can not satisfy global intrinsic order.

Theorem 4. *A hue map does not satisfy global intrinsic order.*

Proof. Any inner point $\gamma(t), 0 < t < 1$ violates the triangle distance difference w.r.t. to the endpoints because the periodicity implies $0 = \Delta E(\gamma(0), \gamma(1))$ but the identity of indiscernibles implies $\Delta E(\gamma(0), \gamma(t)), \Delta E(\gamma(t), \gamma(1)) > 0$. □

Theorem 5. *In a Euclidean color space, a hue map satisfies global intrinsic order if it does not span more than half the circle of hues.*

Proof. In a Euclidean color space, a hue map forms a circle. Let m denote its center. Since two points can not lie further apart than half the circle, they enclose an angle $\angle(x_i, m, x_k) < \pi$. As can be seen in Fig. 4(b), the angles formed with a point between x_i and x_k are smaller because they add up to it $\angle(x_i, m, x_j) + \angle(x_j, m, x_k) = \angle(x_i, m, x_k)$. Then the assertion follows from the law of cosines on an isosceles triangle $c = a\sin(\frac{\gamma}{2})$ and the fact that the sine is monotonically increasing in $[0, \frac{\pi}{2}]$. □

Since local intrinsic order corresponds only to adjacent points, we can assume without loss of generality that they do not lie further apart than half the circle. Therefore Theorem 4 implies the following Corollary.

Corollary 1. *In a Euclidean color space, a hue map satisfies local intrinsic order.*

The hue map suffices local order in any color space known to the authors, but for the case of a path-connected metric space or even a path metric space in which all distances are purely based on experiments, we cannot prove that the hue map or its parts are intrinsically ordered.

Table 1: Summary of the hypotheses, the references where they were stated positively or as negations, and their derived validity w.r.t. to the different types of order within this paper. Green indicates *true* in path-connected metric spaces, cyan indicates *true* in path metric spaces, blue indicates *true* in Euclidean spaces, and red indicates *false*. For consistency and improved coherence, we formulated all hypotheses positively.

Hypothesis	Stated positively	Stated negatively	Legend-based		Intrinsic	
			Local	Global	Local	Global
luminance map is ordered	[5,11,18–20,27,29]		Thm 2	Thm 2	Thm 3	Thm 3
saturation map is ordered	[11,29]		Thm 2	Thm 2	Thm 3	Thm 3
hue map is ordered	[5,11,28]	[5,11,12,18–20,28,31]	Thm 2	Thm 2	Thm 1	Thm 4
monotonicity in luminance is necessary	[18,19]		Sec 4.5	Sec 4.5	Sec 4.5	Sec 4.5
monotonicity in saturation is necessary		[12,18,19,32]	Sec 4.5	Sec 4.5	Sec 4.5	Sec 4.5
monotonicity in hue is necessary			Sec 4.5	Sec 4.5	Sec 4.5	Sec 4.5
monotonicity in luminance is sufficient	[4,16,32]	[15,18,19]	Thm 1	Thm 1	Thm 6	Thm 6
monotonicity in saturation is sufficient			Thm 1	Thm 1	Thm 6	Thm 6
monotonicity in hue is sufficient		[15,18,19]	Thm 1	Thm 1	Thm 6	Thm 6
monotonicity in luminance, RGB, and their order is sufficient	[18,19]	[15]	Thm 1	Thm 1	Thm 7	Thm 7
monotonicity in luminance, RGB, their order, hue, and saturation is sufficient	[15]		Thm 1	Thm 1	Thm 8	Thm 8

4.5 Necessity of Monotonicity

Monotonicity in a specific attribute is not necessary to guarantee order. In particular, strict monotonicity in luminance is not necessary because a saturation map which has constant luminance suffices legend-based order (Theorem 2) and intrinsic order for path metric spaces (Theorem 3). Vice versa, strict monotonicity in luminance is not necessary because the grayscale, which has constant zero saturation, is a counter example. Finally, either map serves as a counter example for monotonicity in hue to be necessary.

4.6 Monotonicity and Intrinsic Order

We designed three counter example color maps to show that monotonicity in hue, saturation, or luminance is not sufficient to guarantee intrinsic order in such a way that they violate the triangle distance difference in the most common color spaces. Please note though that mathematically, it is possible for a path-connected metric space to exist in which the counter examples do not hold.

Theorem 6. *Monotonicity in one attribute: hue, saturation, luminance, is not sufficient to guarantee local or global intrinsic order.*

Proof. We will prove all three assertions by providing a counter example for each. The green colormap in Fig. 1 (left) is monotonic in hue, smoothly transitioning from yellow through green to cyan but because its luminance increases linearly in the first half and decreases in the second half, it does not suffice local intrinsic order at the non-smooth center. Analogously, the orange colormap in Fig. 1 (center) is monotonic in saturation, smoothly transitioning from completely desaturated to fully saturated. However, because its hue goes from red to yellow in the first half and back to red in the second half, it does not suffice local intrinsic order at the non-smooth center. Finally, the purple colormap in Fig. 1 (right) is monotonic in luminance but because its saturation becomes zero in the middle increasing to both sides, it does not suffice local intrinsic order at the non-smooth center.

For each counter example, Fig. 1(bottom) shows negative triangle distance difference w.r.t. CIELAB76, CIELAB00, and CIEUCS02 (triangle distance difference visualized using [9]).

In all three cases the assertion for local intrinsic order implies the one for global intrinsic order, too. □

For the more specific hypotheses regarding "optimal color scales" by Pitzer and Levkovitz [15, 18, 19], we can reuse earlier results.

Theorem 7. *Monotonicity in luminance, RGB, and their order is not sufficient for intrinsic order.*

Proof. We designed the colormap from Fig. 1(right) not only to be monotonic in luminance but also in RGB and their mutual. Therefore the assertion follows from Theorem 6. □

Theorem 8. *Monotonicity in luminance, RGB, and their order, and hue is sufficient for intrinsic order.*

Proof. A colormap that is monotonic in hue is a partial hue map as treated in Subsection 4.4. The restriction w.r.t. RGB ensures that it covers only $\frac{1}{6}$-th of the full circle. For a Euclidean setting, the assertion follows from Theorem 5. □

5 Conclusion

In this paper, we have collected hypotheses from colormapping literature about what is necessary or sufficient to imply perceptual order. Table 1 summarizes a schematic overview with the corresponding references and how we were able to prove or disprove their validity.

Our overall findings can be summarized as follows:

- Monotonicity in any attribute is sufficient to imply legend-based order in any path-connected metric space.
- There is no single attribute in which monotonicity is necessary to imply order.
- Luminance and saturation are intrinsically ordered in a path metric space.
- A hue map is generally not intrinsically ordered, but it is in the special case of a Euclidean space if it does not exceed one half of the full hue circle.

It is far easier to test for monotonicity than for order. In future work, a more complete treatment of order will require perceptual user evaluation and understanding the interplay of data and task with perceptual order. Yet we hope that by using these current results, visualization practitioners will be able to design colormaps to meet their data and task specific needs more effectively.

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